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Internal Model Control of Fully-actuated Discrete Uncertain Systems

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Abstract— **An approach of Internal Model Control (IMC) of linear multivariable (MIMO) sampled Uncertain systems is proposed in this paper. The latter discusses the robustness of such sampled system with parametric uncertainty using Kharitonov's theorem and Jury stability criterion. An application is then presented to show the reliability of the proposed design approach by ensuring stability and rejecting disturbances.**

Keywords— **Internal Model Control, MIMO systems, Fully actuated systems, Stability, Uncertain systems, Instable zero, Disturbances rejection.**

I. Introduction

Industrial systems are very frequently multivariable. They have more than one control input and more than one output. Depending on their number, there are three classes of systems, namely: Fully actuated system (square system is a system having the same number for inputs and outputs), Over-actuated system (non-square system that their number of inputs is superior than that of outputs) and Under-actuated system (a system where the number of inputs is inferior than the number of outputs) [1, 2, 3, 6, 12]. In this work, we are interested in systems having the same number of input-outputs and functionally controllable.

The objective of the control is then to obtain a desirable behaviour of several outputs variables simultaneously, by the manipulation of several inputs. The realization of these control law is based on the modelling of systems.

Nevertheless the model, the regulator are generally and initially given in a continuous time model in the form of a transfer function matrix for multivariable systems, but in some experimental applications, we need to discretise the continuous time model. Then, we distinguish different discretization techniques to convert continuous systems into discrete systems such as impulse invariant method, bilinear transformation (Tustin transformation), state-transition method, … [1, 4, 5]. In this paper, we are interested in the Impulse Invariant Method discretization which produces a discrete time model in such a way that the impulse response is the same (invariant) at the sampling instants.

The calculation of any physical process control requires necessarily a model which can never be a perfect representation of reality: there are always uncertainties of modelling, whose consequence is that the behaviour of a physical system cannot be described exactly by a mathematical model [7, 8]. Indeed, there are different approaches that are proposed for the synthesis of the control law for uncertain multivariable systems such as the control of the internal model that was introduced by Garcia and Morari in 1982 and supplemented by a series of publications by these same authors [9, 10, 11].

The IMC is a powerful controller design strategy for linear systems, using a process model controlled by the same control input applied to the process. It is exploited in the industrial systems by these robustness advantages, the simplicity of construction and the compensation of the errors of modelling.

The objective of this work is the application of the internal model control of a class of multivariable uncertain discrete systems that is an extension of the IMC structure applied in [1]. This paper is organised as follows. Section II is dedicated to the design of the internal model control of multivariable discrete system. Section III presents the notion of uncertain systems and defines the kharitonov theorem that allows us to study the stability of uncertain systems. In Section IV, an application is applied to an uncertain discrete multivariable system to show the robustness and validity of this proposed design which provides stability and preserves system performances despite parametric uncertainties and external disturbances.

II. IMC of Multivariable Discret Systems

A. Proposed IMC Structure for Multivariable Systems

The Internal Model Control (IMC) takes up the basic principle of the open loop control, which represents a major interest for stability, integrating the advantages of the closed loop allowing the rejection of disturbances and modelling errors.

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The IMC incorporates a simulation of the process by an internal model in its control structure. Its application mainly concerns the stable systems in open loop. The difference which can exist between the outputs of the process and its model is brought back at the entrance of the control block. The regulator, obtained like reverse approximated of the model, acts simultaneously on the process and its model in order to compensate for this variation [10, 13].

The IMC structure of a multivariate system, having m_inputs - m _outputs can be schematized as shown in Figure 1:

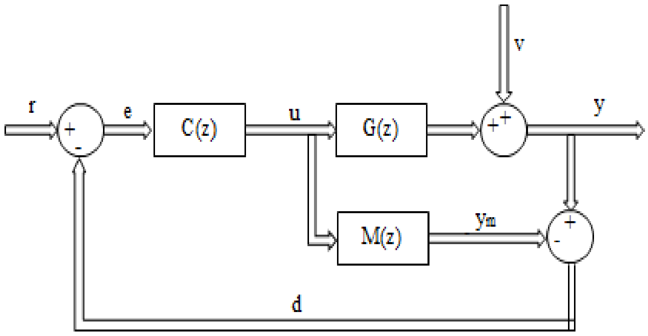

Fig. 1 Basic IMC structure of multivariable system

The configuration of the IMC controller for multivariable systems is represented by:

- G(z) and M(z): are respectively the matrices of transfer of multivariable system and its model of dimension (m x m).
- C(z): the transfer matrix controller of dimension (m x m).
- y and y_m: present respectively the output vectors of the process and the model of dimension (m x 1) .
- v: the perturbation vector and of dimension (m x 1).
- d: present the difference between the output and its model.
- r: is the reference vector of dimension (m x 1).

This control structure is defined by the following equations:

$$d = y - y_m = (G - M)u + v \quad (1)$$

$$u = \frac{C}{I_m + C(G-M)}(r - v) \quad (2)$$

$$y = \frac{GC}{I_m + C(G-M)} r + \frac{I_m - CM}{I_m + C(G-M)} v \quad (3)$$

with I_m is the identity matrix.

The synthesis of an IMC corrector that is equal to the direct inverse of the model is essential in order to ensure a perfect follow-up of the reference instructions.
Then, an inversion method has been proposed in [14, 13] to obtain the following IMC regulator:

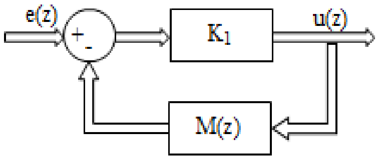

Fig. 2 Blocks of the inversion proposed in the multivariable case

with:

K_1 : is a square matrix of inversion of dimension (m x m)
The inversion matrix K_1 is an invertible square matrix. It must provide regulator stability. We can choose K_1 of the form:

$$K_1 = a \times I_m \quad (4)$$

such as $a \in \mathbb{R}^+$

From Figure 2, the IMC regulator transfer matrix can be expressed by the following expression:

$$C(z) = \frac{u(z)}{e(z)} = \frac{K_1}{I_m + K_1 M(z)} = \frac{1}{K_1^{-1} + M(z)} \quad (5)$$

In order to approximate the controller transfer function C(z) to $M(z)^{-1}$, we should just select the gain a sufficiently high. So, we obtain:

$$C(z) \simeq M(z)^{-1} \quad (6)$$

The matrix of the static gains of the regulator C(1) can be expressed according to the matrix of the static gains of the system M(1) . It is defined by the equation:

$$C(1) = \frac{K_1}{I_m + K_1 M(1)} = \left(K_1^{-1} + M(1)\right)^{-1} \quad (7)$$

However for certain cases, for example for systems with instable zero or time-delay, the coefficient K_1 ensuring the stability of the regulator cannot be chosen sufficiently high. What can involve the degradation of the precision of the system (i.e. the static error is non-null). In order to treat such problem, a second structure CMI can be proposed for this class of dynamic systems by adding a second gain matrix K_2 which aims to compensate for the static errors of the multivariable system as shown in figure 3.

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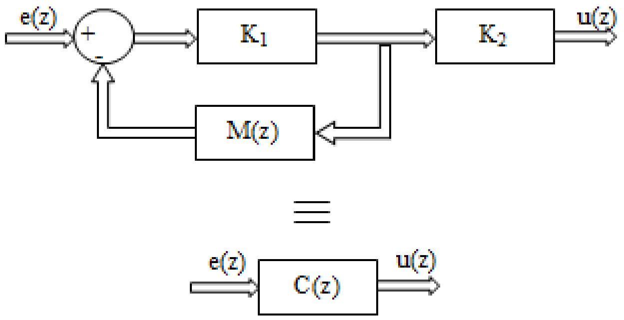

Fig. 3 Generalized multivariable controller structure

K_2 can be written in the following form:

$$K_2 = K_1^{-1}(I_m + K_1 M(1)) M(1)^{-1} \quad (8)$$

We can then express the control vector C(z) in the following form:

$$C(z) = K_2 (I_m + K_1 M(z))^{-1} K_1 \quad (9)$$

The stability of the proposed controller given in (9) depends on the stability of the model and a best choice of K_1.

B. *IMC's Stability*

We can note that the output vector can be written in the following form in order to replace C(z) by its expression given in (9):

$$y(z) = y_r(z) r(z) + y_v(z) v(z) \quad (10)$$

with:

$$y_r = \frac{G K_2 K_1}{I_m (I_m + K_1 M) + K_1 K_2 (G - M)} \quad (10a)$$

$$y_v = \frac{I_m (I_m + K_1 M) - K_1 K_2 M}{I_m (I_m + K_1 M) + K_1 K_2 (G - M)} \quad (10b)$$

The stability of IMC structure depends on the stability of the process to be controlled, its model and the proposed regulator. So, to ensure the stability of the system, it is necessary that each block of the IMC structure is stable in open loop. Given a stable process and model, controller stability only depends on K_1.

III. UNCERTAIN DISCRET SYSTEMS

Generally, uncertainties are grouped into two categories, structured uncertainties and unstructured uncertainties. The first type is often called parametric uncertainty and the second dynamic uncertainty.

The structured uncertainties may be represented by variations of certain physical system parameters over some possible value ranges (complex or real). This type of uncertainties is due to the fact that the parameters could not be accurately modelled or measured. They affect the low-frequency range performance.

Unstructured uncertainties are the ones that affect the system even without having any structural information; this might be due to high frequencies and dynamic uncertainties. Generally, this type of uncertainty can be represented as additive by an unknown transfer function matrix (input multiplicative or output multiplicative). In this work, we are interested of a linear system with parametric uncertainties [7, 8].

We consider in this paper a multivariable linear system given by a transfer function matrix G(z) such as:

$$G(z) = G_{ij}(z) \; ; \begin{cases} i = 1, 2, \ldots, m \\ j = 1, 2, \ldots, m \end{cases} \quad (11)$$

If we suppose the presence of parametric uncertainties, $G_{ij}(z)$ is written in the following form:

$$G_{ij}(z) = \frac{N_{ij}(z)}{D_{ij}(z)} = \frac{b_m z^m + b_{m-1} z^{m-1} + \ldots + b_0}{a_n z^n + a_{n-1} z^{n-1} + \ldots + a_0} \quad (12)$$

with:

$$n > m; \begin{cases} b_m \in [b_m^-, b_m^+] \\ b_{m-1} \in [b_{m-1}^-, b_{m-1}^+] \\ . \\ . \\ b_0 \in [b_0^-, b_0^+] \end{cases} \quad and \quad \begin{cases} a_n \in [a_n^-, a_n^+] \\ a_{n-1} \in [a_{n-1}^-, a_{n-1}^+] \\ . \\ . \\ a_0 \in [a_0^-, a_0^+] \end{cases}$$

A. *Uncertain System's Stability*

There are different methods to study the robust stability of the system depending on the type of transfer function parameters. If the coefficients are time-invariant, we can use the Jury stability criterion to check the discrete system stability. But, if there is uncertainty concerning the parameters of the transfer function, it is necessary to use Kharitonov's theorem to study the linear system stability [7, 15].

Kharitonov's theorem:

Kharitonov's theorem allowed characterizing the stability of a system subjected to bounded parametric uncertainties on the transfer function coefficients.

Let the characteristic polynomial P be described by the following equation:

$$P(z) = l_0 + l_1 z + l_2 z^2 + \ldots + l_n z^n \quad (13)$$

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Where $l_i \in \mathbb{R}$ are known only in specified ranges such as $l_i^- < l_i < l_i^+$ for i=1; 2 ; … ; n.

The polynomials family P is stable if and only if the following four Kharitonov polynomials (P_1 , P_2 , P_3 and P_4) are stable [15]:

$$\begin{cases} P_1(z) = l_0^- + l_1^- z + l_2^+ z^2 + l_3^+ z^3 + l_4^- z^4 + \dots \\ P_2(z) = l_0^- + l_1^+ z + l_2^+ z^2 + l_3^- z^3 + l_4^- z^4 + \dots \\ P_3(z) = l_0^+ + l_1^- z + l_2^- z^2 + l_3^+ z^3 + l_4^+ z^4 + \dots \\ P_4(z) = l_0^+ + l_1^+ z + l_2^- z^2 + l_3^- z^3 + l_4^+ z^4 + \dots \end{cases} \quad (14)$$

We then obtain the Kharitonov polynomials corresponding to D:

$$\begin{cases} D_1(z) = a_0^- + a_1^- z + a_2^+ z^2 + a_3^+ z^3 + a_4^- z^4 + \dots \\ D_2(z) = a_0^- + a_1^+ z + a_2^+ z^2 + a_3^- z^3 + a_4^- z^4 + \dots \\ D_3(z) = a_0^+ + a_1^- z + a_2^- z^2 + a_3^+ z^3 + a_4^+ z^4 + \dots \\ D_4(z) = a_0^+ + a_1^+ z + a_2^- z^2 + a_3^- z^3 + a_4^+ z^4 + \dots \end{cases} \quad (15)$$

B. *Uncertain System's Control*

The main aim of the robust control for an uncertain system is to guarantee the performances and the stability of a system despite of the risks and fluctuations which can affect the system during its operation.

There exists a difference between the observed behaviour of the real system and its nominal model. The main problem of this type of control is the control law synthesis in closed loop, to guarantee the imposed performance despite model imperfections, uncertainties and external disturbances.

IV. APPLICATION

Considering the transfer matrix G(z) of the uncertain multivariable process with two inputs-two outputs. Uncertainties occur at the level of the first element of this matrix $G_{11}(z)$. It is defined by the following transfer matrix:

$$G(z) = Z\left[B_0(p) \begin{pmatrix} \frac{b_1 p + b_0}{p^2 + a_1 p + a_0} & \frac{1}{p+2} \\ \frac{p+1}{4p^2 + 3p + 2} & \frac{3}{p+4} \end{pmatrix} \right] \quad (16)$$

With:

$$b_0 \in [b_{0\min}, b_{0\max}] = [1, 4]$$
$$b_1 \in [b_{1\min}, b_{1\max}] = [1, 2]$$
$$a_0 \in [a_{0\min}, a_{0\max}] = [1, 3]$$
$$a_1 \in [a_{1\min}, a_{1\max}] = [1, 4]$$

This uncertain system is represented by the following nominal model:

$$M(z) = Z\left[B_0(p) \begin{pmatrix} \frac{1.5p + 2.5}{p^2 + 2.5p + 2} & \frac{1}{p+2} \\ \frac{p+1}{4p^2 + 3p + 2} & \frac{3}{p+4} \end{pmatrix} \right] \quad (17)$$

A. *Application of the IMC without K_2*

Firstly, let's consider the case characterized by the absence of disturbances.
For a reference vector that is chosen as steps of amplitude 1 applied at t=0s , the simulation results for a gain K_1=2 and a sampling time T_e=0.1s are given in figures 4 and 5.

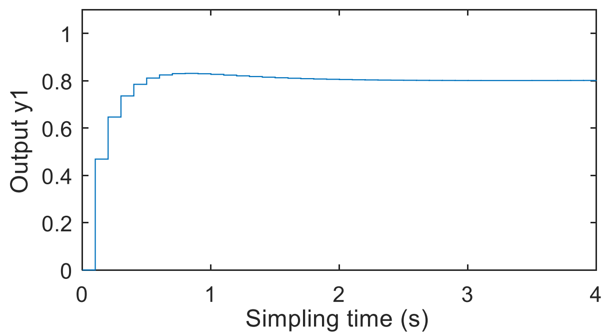

Fig. 4 Output signal y_1 without K_2

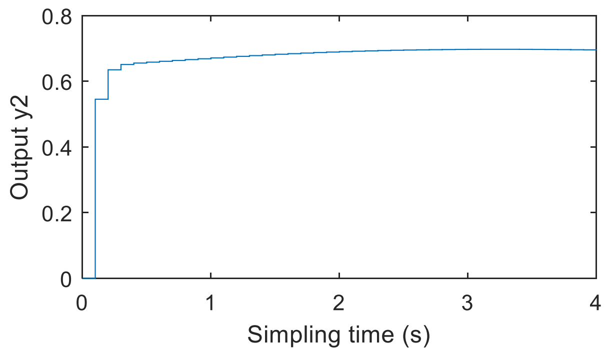

Fig. 5 Output signal y_2 without K_2

It can be remark from these two figures that the internal model control applied maintains the stability of the chosen discrete multivariable system despite the absence of the gain K_2 and the presence of uncertainties. Then, K_1 gain is responsible for maintaining stability. But, it is clear that the outputs y_1 and y_2 do not describe perfectly the reference signals.

B. *Application of the IMC with K_2*

Let's consider now the presence of the gain K_2 where K_2 defined by (8). For the same reference and a sampling period T_e=0.1s and K_1=2, the figures 6 and 7 present the simulation results:

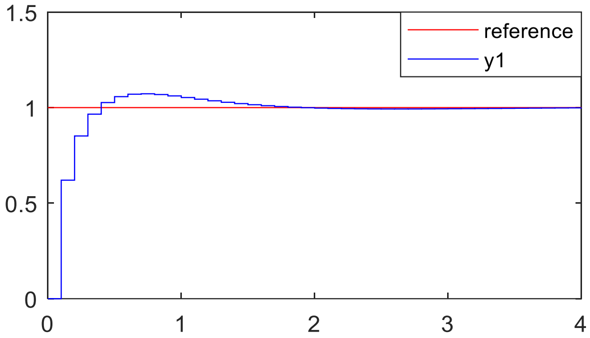

Fig. 6 Output signal y_1 with K_2

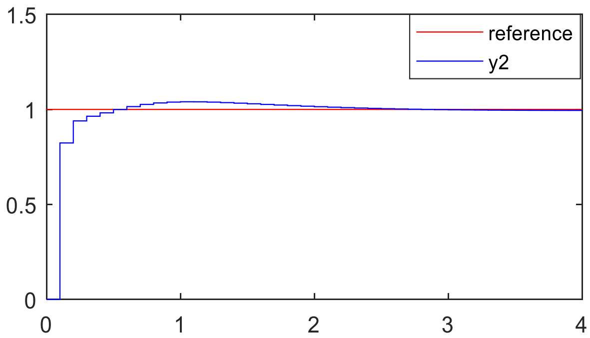

Fig. 7 Output signal y_2 with K_2

We compare Figures 4 and 5 with Figures 6 and 7, we remark that the outputs of the system perfectly reach the input reference when adding K_2; the added gain K_2 then is able to compensate for the static error as desired. In this way, we can see that the system remains stable despite the presence of uncertainties.

For the same references and the regulator used previously, we must change the sampling period now.

For T_e=0.2s, the simulation results are given in figures 8 and 9.

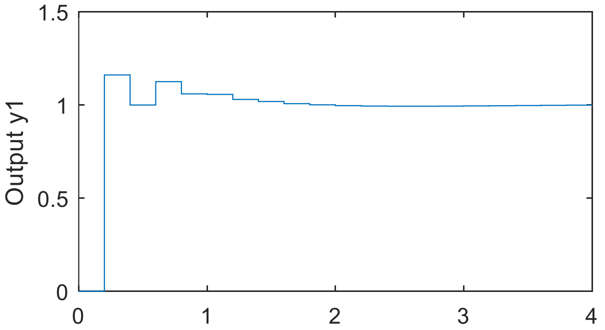

Fig. 8 Output signal y_1 for T_e=0.2s

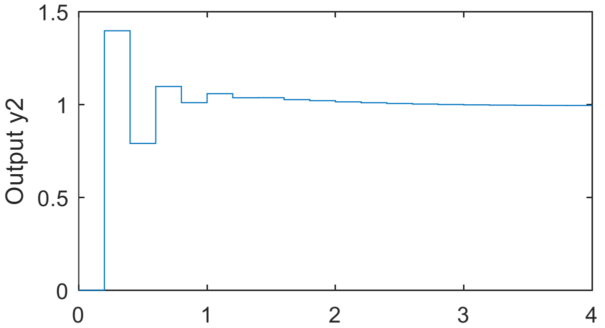

Fig. 9 Output signal y_2 for T_e=0.2s

For T_e=0.3s, the simulation results are given in figures 10 and 11.

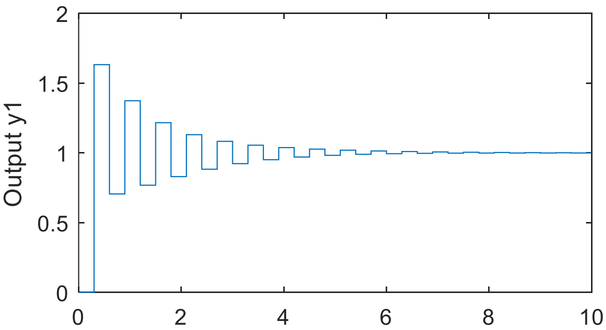

Fig. 10 Output signal y_1 for T_e=0.3s

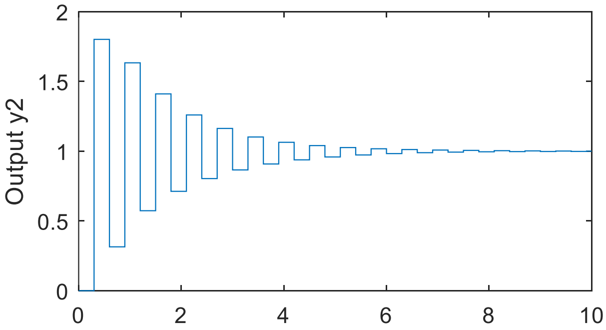

Fig. 11 Output signal y_2 for T_e =0.3s

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From Figures 8 - 11, it can be shown that as for the same value of gains K_1 and K_2 , a high value of the sampling time can result in deterioration of closed loop system performances.

C. Application of the IMC with disturbances

Let's consider now the presence of external disturbances in the form of a unit steps. And, let's show its effect in the case of the internal model control proposed. The simulations results are given by the figures 12 and 13.

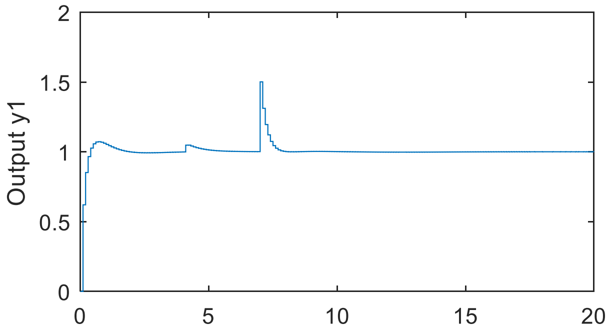

Fig. 12 Output signal y_1 with disturbances

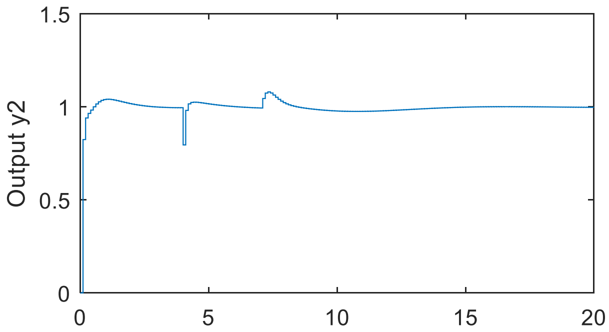

Fig. 13 Output signal y_2 with disturbances

Significant peaks of the input signals appear at the moment already chosen for the disturbances, it is clear that the discrete uncertain system controlled by IMC is able to maintain stability despite external disturbances.

The figures show a robust behaviour even on the presence of disturbances directly affecting the outputs of the process.

V. CONCLUSIONS

In this paper an approach for IMC of linear fully actuated discrete uncertain systems is developed. This approach uses an internal model that is nominal of the uncertain system. The chosen system is a two-input-two-output linear system.

Satisfactory results have been obtained showing the robustness of this approach to maintain stability and reject external disturbances.

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